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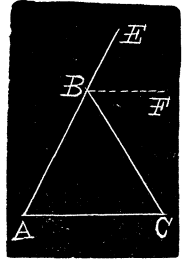
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$$\angle EBC = \angle A + \angle C = 2\angle C.$$

But $\angle EBC = 2\angle FBC$, since BF is the bisector of $\angle EBC$. $\therefore 2\angle FBC = 2\angle C$, or $\angle FBC = \angle C$.

Hence, BF is parallel to AC , because if two straight lines are cut by a third straight making the alternate interior angles equal, the two lines are parallel.



Solutions were also received from *J. K. ELLWOOD*, *H. G. WHITAKER*, and *G. B. M. JERR*.

NOTE—No solution has yet been received to problem 20.

CALCULUS.

Conducted by *J. M. COLAW*, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

20. Proposed by *F. P. MATZ*, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

$$\int_0^{\frac{1}{2}\pi} \sqrt{(1-e^2 \cos^2 \phi)(1-e^2 \sin^2 \phi)} d\phi = \text{what?}$$

Solution by Professor *J. F. W. SCHEFFER*, A. M., Hagerstown, Maryland.

$$\begin{aligned} \sqrt{(1-e^2 \cos^2 \phi)(1-e^2 \sin^2 \phi)} &= \sqrt{1-e^2(\cos^2 \phi + \sin^2 \phi) + e^4 \sin^2 \phi \cos^2 \phi} \\ &= \sqrt{1-e^2 + \frac{e^4}{4} \sin^2 2\phi} = \sqrt{1-e^2 + \frac{e^2}{4} - \frac{e^4}{4} \cos^2 2\phi} \\ &= \frac{1}{2} \sqrt{(2-e^2)^2 - e^4 \cos^2 2\phi} = \frac{1}{2} \sqrt{(2-e^2)^2 - e^4 \sin^2 \left(\frac{\pi}{2} - 2\phi\right)} \\ &= \frac{1}{2} (2-e^2)^2 \sqrt{1 - \frac{e^4}{(2-e^2)^2} \sin^2 \left(\frac{\pi}{2} - 2\phi\right)}. \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{2}} d\phi \sqrt{(1-e^2 \cos^2 \phi)(1-e^2 \sin^2 \phi)} \\ &= -\frac{1}{4} (2-e^2)^2 \int_{\frac{1}{2}\pi}^{-\frac{1}{2}\pi} d\left(\frac{\pi}{2} - 2\phi\right) \sqrt{1 - \frac{e^4}{(2-e^2)^2} \sin^2 \left(\frac{\pi}{2} - 2\phi\right)} \\ &= \frac{1}{4} (2-e^2)^2 \int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi} d\lambda \sqrt{1 - \frac{e^4}{(2-e^2)^2} \sin^2 \lambda} \end{aligned}$$

$$= \frac{1}{2}(2-e^2) \int_0^{\frac{\pi}{2}} dx \sqrt{1 - \frac{e^4}{(2-e^2)^2} \sin^2 \lambda} = \frac{1}{2}(2-e^2) E\left(\frac{e^2}{2-e^2}, \frac{\pi}{2}\right) = \frac{1}{2}, \text{ if } e=1.$$

Also solved by OTTO GEUKELER, WILLIAM HOOVER, ARTEMAS MARTIN, F. P. MATZ, and G. B. M. ZERR.

21. Proposed by T. JOHN COLE, Columbus, Ohio.

In the equilateral triangle ABC , AB the base is 10 feet. With B as a center an arc is drawn from C to A ; likewise with A as a center an arc is drawn from C to B . What is the volume of the solid generated by revolving the figure about the altitude of the triangle as an axis.

I. Solution by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

The triangle revolving describes a cone the volume of which is $\frac{1}{3}\pi \cdot 25 \cdot 5\sqrt{3} = 226.725$.

The area of either segment is $50\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) = 9.05861$; the distance of the center of gravity of the segment from the center of the circle $= \frac{1000}{12 \text{ Area}}$, and from the axis of revolution is $\left(\frac{1000}{12 \text{ Area}} - 5 \sec 30^\circ\right) \cos 30^\circ = 2.9669$.

The product of the area by the path of the center of gravity $9.05861 \times 2\pi \cdot 2.9669 = 168.865$.

Adding to this the volume of the cone, the answer is 395.59 cubic feet.

II. Solution by JOHN DOLMAN, Jr., Counsellor at Law, Philadelphia Pennsylvania.

Let the origin be at the centre of the base. Put the side of the triangle $= 2r$, and let y = the radius of any circular horizontal section at any variable height x .

Then $V = \int_0^{r\sqrt{3}} \pi y^2 dx \dots (1)$. The equation of the arc forming the side is $4r^2 = x^2 + (y+r)^2$ from which $y^2 = 5r^2 - x^2 - 2r\sqrt{(4r^2 - x^2)}$ which value of y^2 substituted in (1) gives $V = \pi \int_0^{r\sqrt{3}} [(5r^2 - x^2 - 2r\sqrt{(4r^2 - x^2)})] dx$

$$= \pi \left[5r^2 x - \frac{1}{3} x^3 - r \left[x\sqrt{(4r^2 - x^2)} + 4r^2 \sin^{-1} \frac{x}{2r} \right] \right]_0^{r\sqrt{3}}$$

$= \pi(3r^3\sqrt{3} - \frac{4}{3}\pi r^3) = \frac{\pi r^3}{3}(9\sqrt{3} - 4\pi) = 3.164722r^3$, and putting $r=5$ this gives $V = 395.59 +$ cubic feet.

III. Solution by OTTO GEUKELER, Bloomington, Indiana.

The equation of circle with O as origin is $y^2 + (x-5)^2 = 100$, or $x=5$